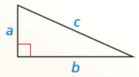
Theorem

Theorem 9.1 Pythagorean Theorem

In a right triangle, the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the legs.



$c^2 = a^2 + b^2$

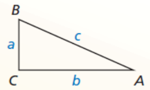
Proof Explorations 1 and 2, p. 463; Ex. 39, p. 484

Theorem

Theorem 9.2 Converse of the Pythagorean Theorem

If the square of the length of the longest side of a triangle is equal to the sum of the squares of the lengths of the other two sides, then the triangle is a right triangle.

If $c^2 = a^2 + b^2$, then $\triangle ABC$ is a right triangle.



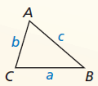
Proof Ex. 39, p. 470

Theorem

Theorem 9.3 Pythagorean Inequalities Theorem

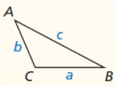
For any $\triangle ABC$, where c is the length of the longest side, the following statements are true.

If $c^2 < a^2 + b^2$, then $\triangle ABC$ is acute.



$c^2 < a^2 + b^2$

If $c^2 > a^2 + b^2$, then $\triangle ABC$ is obtuse.



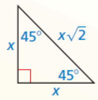
$c^2 > a^2 + b^2$

Proof Exs. 42 and 43, p. 470

Theorem

Theorem 9.4 45°-45°-90° Triangle Theorem

In a 45°-45°-90° triangle, the hypotenuse is $\sqrt{2}$ times as long as each leg.



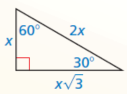
hypotenuse = leg • $\sqrt{2}$

Proof Ex. 19, p. 476

Theorem

Theorem 9.5 30°-60°-90° Triangle Theorem

In a 30°-60°-90° triangle, the hypotenuse is twice as long as the shorter leg, and the longer leg is $\sqrt{3}$ times as long as the shorter leg.



hypotenuse = shorter leg • 2

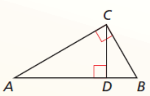
longer leg = shorter leg • $\sqrt{3}$

Proof Ex. 21, p. 476

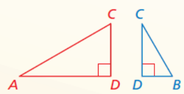
Theorem

Theorem 9.6 Right Triangle Similarity Theorem

If the altitude is drawn to the hypotenuse of a right triangle, then the two triangles formed are similar to the original triangle and to each other.



$\triangle CBD \sim \triangle ABC$, $\triangle ACD \sim \triangle ABC$, and $\triangle CBD \sim \triangle ACD$.



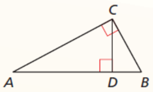
Proof Ex. 45, p. 484

Theorems

Theorem 9.7 Geometric Mean (Altitude) Theorem

In a right triangle, the altitude from the right angle to the hypotenuse divides the hypotenuse into two segments.

The length of the altitude is the geometric mean of the lengths of the two segments of the hypotenuse.



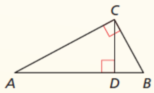
$CD^2 = AD \cdot BD$

Proof Ex. 41, p. 484

Theorem 9.8 Geometric Mean (Leg) Theorem

In a right triangle, the altitude from the right angle to the hypotenuse divides the hypotenuse into two segments.

The length of each leg of the right triangle is the geometric mean of the lengths of the hypotenuse and the segment of the hypotenuse that is adjacent to the leg.



$CB^2 = DB \cdot AB$
$AC^2 = AD \cdot AB$

Proof Ex. 42, p. 484

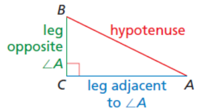
Core Concept

Tangent Ratio

Let $\triangle ABC$ be a right triangle with acute $\angle A$.

The tangent of $\angle A$ (written as tan A) is defined as follows.

$$\tan A = \frac{\text{length of leg opposite } \angle A}{\text{length of leg adjacent to } \angle A} = \frac{BC}{AC}$$



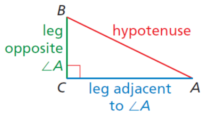
Core Concept

Sine and Cosine Ratios

Let $\triangle ABC$ be a right triangle with acute $\angle A$. The sine of $\angle A$ and cosine of $\angle A$ (written as sin A and cos A) are defined as follows.

$$\sin A = \frac{\text{length of leg opposite } \angle A}{\text{length of hypotenuse}} = \frac{BC}{AB}$$

$$\cos A = \frac{\text{length of leg adjacent to } \angle A}{\text{length of hypotenuse}} = \frac{AC}{AB}$$



Core Concept

Sine and Cosine of Complementary Angles

The sine of an acute angle is equal to the cosine of its complement. The cosine of an acute angle is equal to the sine of its complement.

Let A and B be complementary angles. Then the following statements are true.

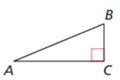
$\sin A = \cos(90° - A) = \cos B$ $\quad$ $\sin B = \cos(90° - B) = \cos A$

$\cos A = \sin(90° - A) = \sin B$ $\quad$ $\cos B = \sin(90° - B) = \sin A$

Core Concept

Inverse Trigonometric Ratios

Let $\angle A$ be an acute angle.



Inverse Tangent If $\tan A = x$, then $\tan^{-1} x = m\angle A$. $\quad \tan^{-1} \frac{BC}{AC} = m\angle A$

Inverse Sine If $\sin A = y$, then $\sin^{-1} y = m\angle A$. $\quad \sin^{-1} \frac{BC}{AB} = m\angle A$

Inverse Cosine If $\cos A = z$, then $\cos^{-1} z = m\angle A$. $\quad \cos^{-1} \frac{AC}{AB} = m\angle A$

Core Concept

Solving a Right Triangle

To **solve a right triangle** means to find all unknown side lengths and angle measures. You can solve a right triangle when you know either of the following.

- two side lengths
- one side length and the measure of one acute angle