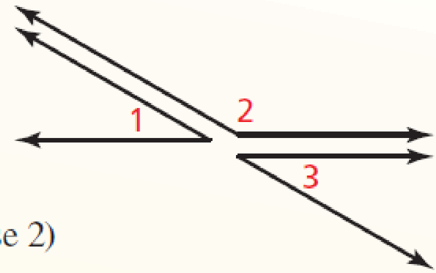
Theorems

Theorem 2.4 Congruent Supplements Theorem

If two angles are supplementary to the same angle (or to congruent angles), then they are congruent.

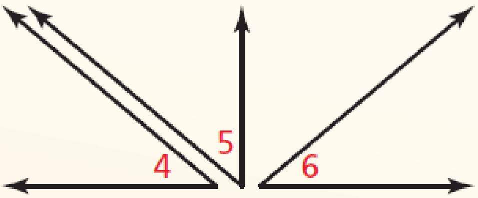


If ∠1 and ∠2 are supplementary and ∠3 and ∠2 are supplementary, then ∠1 ≅ ∠3.

Proof Example 2, p. 107 (case 1); Ex. 20, p. 113 (case 2)

Theorem 2.5 Congruent Complements Theorem

If two angles are complementary to the same angle (or to congruent angles), then they are congruent.



If ∠4 and ∠5 are complementary and ∠6 and ∠5 are complementary, then ∠4 ≅ ∠6.

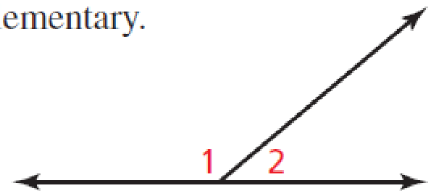
Proof Ex. 19, p. 112 (case 1); Ex. 22, p. 113 (case 2)

Postulate and Theorem

Postulate 2.8 Linear Pair Postulate

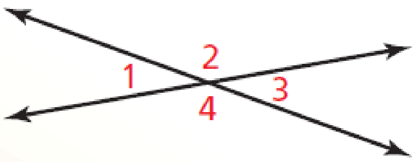
If two angles form a linear pair, then they are supplementary.

$\angle 1$ and $\angle 2$ form a linear pair, so $\angle 1$ and $\angle 2$ are supplementary and $m\angle 1 + m\angle 2 = 180°$.



Theorem 2.6 Vertical Angles Congruence Theorem

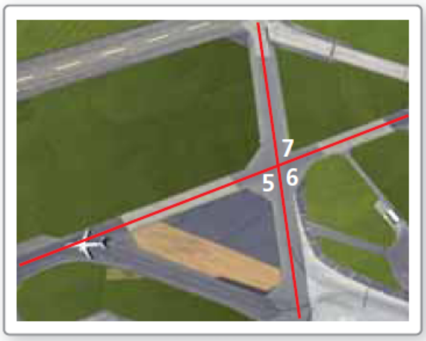
Vertical angles are congruent.



Proof Example 3, p. 108

$\angle 1 \cong \angle 3$, $\angle 2 \cong \angle 4$

Use the given paragraph proof to write a two-column proof of the Vertical Angles Congruence Theorem.



Given ∠5 and ∠7 are vertical angles.

Prove ∠5 ≅ ∠7

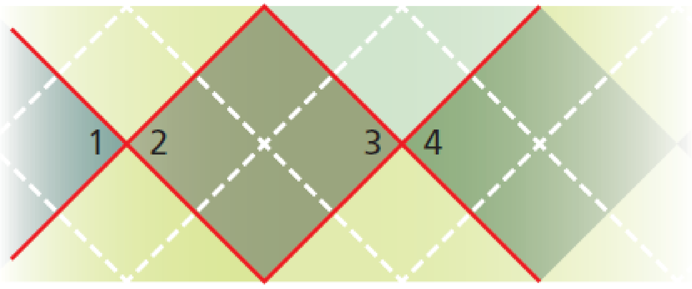
Paragraph Proof

∠5 and ∠7 are vertical angles formed by intersecting lines. As shown in the diagram, ∠5 and ∠6 are a linear pair, and ∠6 and ∠7 are a linear pair. Then, by the Linear Pair Postulate, ∠5 and ∠6 are supplementary and ∠6 and ∠7 are supplementary. So, by the Congruent Supplements Theorem, ∠5 ≅ ∠7.

Write a paragraph proof.

Given ∠1 ≅ ∠4

Prove ∠2 ≅ ∠3

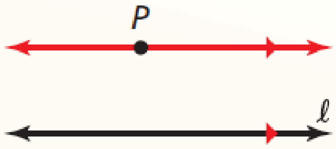


Postulates

Postulate 3.1 Parallel Postulate

If there is a line and a point not on the line, then there is exactly one line through the point parallel to the given line.

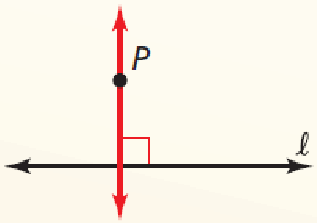
There is exactly one line through P parallel to ℓ.



Postulate 3.2 Perpendicular Postulate

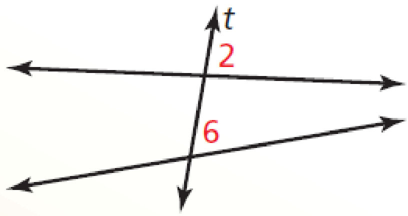
If there is a line and a point not on the line, then there is exactly one line through the point perpendicular to the given line.

There is exactly one line through P perpendicular to ℓ.

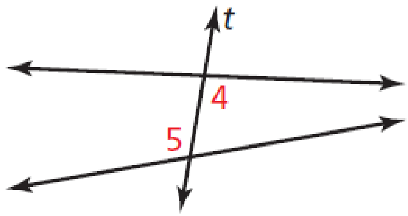


Core Concept

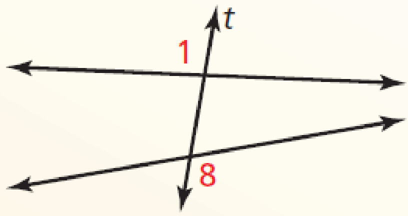
Angles Formed by Transversals



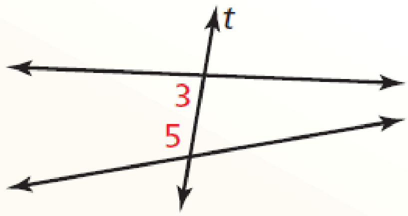
Two angles are **corresponding angles** when they have corresponding positions. For example, ∠2 and ∠6 are above the lines and to the right of the transversal *t*.



Two angles are **alternate interior angles** when they lie between the two lines and on opposite sides of the transversal *t*.



Two angles are **alternate exterior angles** when they lie outside the two lines and on opposite sides of the transversal *t*.



Two angles are **consecutive interior angles** when they lie between the two lines and on the same side of the transversal *t*.

Theorems

Theorem 3.1 Corresponding Angles Theorem

If two parallel lines are cut by a transversal, then the pairs of corresponding angles are congruent.

Examples In the diagram at the left, $\angle 2 \cong \angle 6$ and $\angle 3 \cong \angle 7$.

Proof Ex. 36, p. 180

Theorem 3.2 Alternate Interior Angles Theorem

If two parallel lines are cut by a transversal, then the pairs of alternate interior angles are congruent.

Examples In the diagram at the left, $\angle 3 \cong \angle 6$ and $\angle 4 \cong \angle 5$.

Proof Example 4, p. 134

Theorem 3.3 Alternate Exterior Angles Theorem

If two parallel lines are cut by a transversal, then the pairs of alternate exterior angles are congruent.

Examples In the diagram at the left, $\angle 1 \cong \angle 8$ and $\angle 2 \cong \angle 7$.

Proof Ex. 15, p. 136

Theorem 3.4 Consecutive Interior Angles Theorem

If two parallel lines are cut by a transversal, then the pairs of consecutive interior angles are supplementary.

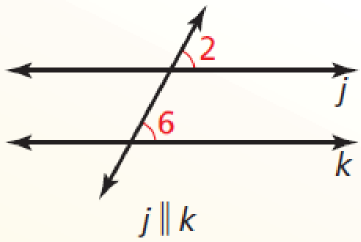
Examples In the diagram at the left, $\angle 3$ and $\angle 5$ are supplementary, and $\angle 4$ and $\angle 6$ are supplementary.

Proof Ex. 16, p. 136

Theorem

Theorem 3.5 Corresponding Angles Converse

If two lines are cut by a transversal so the corresponding angles are congruent, then the lines are parallel.

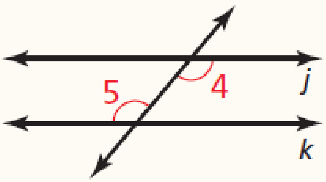


Proof Ex. 36, p. 180

Theorems

Theorem 3.6 Alternate Interior Angles Converse

If two lines are cut by a transversal so the alternate interior angles are congruent, then the lines are parallel.

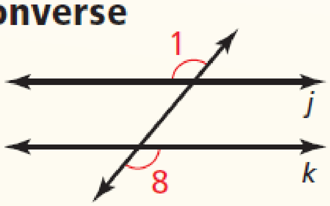


$j \parallel k$

Proof Example 2, p. 140

Theorem 3.7 Alternate Exterior Angles Converse

If two lines are cut by a transversal so the alternate exterior angles are congruent, then the lines are parallel.

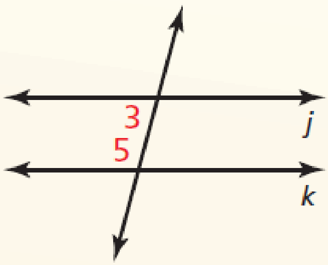


$j \parallel k$

Proof Ex. 11, p. 142

Theorem 3.8 Consecutive Interior Angles Converse

If two lines are cut by a transversal so the consecutive interior angles are supplementary, then the lines are parallel.



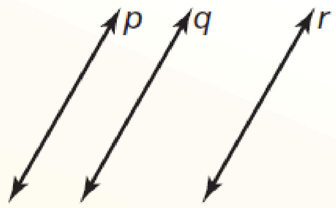
If $\angle 3$ and $\angle 5$ are supplementary, then $j \parallel k$.

Proof Ex. 12, p. 142

Theorem

Theorem 3.9 Transitive Property of Parallel Lines

If two lines are parallel to the same line, then they are parallel to each other.



If $p \parallel q$ and $q \parallel r$, then $p \parallel r$.

Proof Ex. 39, p. 144; Ex. 48, p. 162

Each step is parallel to the step immediately above it. The bottom step is parallel to the ground. Explain why the top step is parallel to the ground.

In the diagram below, $p \parallel q$ and $q \parallel r$. Find $m\angle 8$. Explain your reasoning.