Algebra I Notes – Lesson 6.4/6.5B: More Multiplication & Division Properties of Exponents

Objectives: Students will be able to raise a power to a power, a power of a product and a power of a quotient.

Powers of Powers:

Original Problem: $(3^6)^2$ = **?**

Expanded Problem: $= 3^6 \bullet 3^6$

Simplified in exponential form: $= 3^{12}$

Question: What is the rule for simplifying a power of a power?

Raising a Power to a Power: $(x^m)^n = x^{m \cdot n}$

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Example 1: Raising a power to a power

a. $(5^4)^3$

$5^4 \cdot 5^4 \cdot 5^4$

5^{12}

$5^{4\cdot 3} = 5^{12}$

b. $(2^7)^5$

$2^7 \cdot 2^7 \cdot 2^7 \cdot 2^7 \cdot 2^7$

2^{35}

$2^{7\cdot 5} = 2^{35}$

c. $(2^2)^8$

$2^{2\cdot 8}$

2^{16}

Example 2: Raising a power to a power in an Algebraic Expression

a. $(g^4)^3$

g^{12}

b. $(c^9)^{-3}$

$c^{-27} = \frac{1}{c^{27}}$

c. $(b^{-3})^{-2}$

b^6

d. $(k^4)^3 \bullet k^5$

$k^{12} \cdot k^5$

k^{17}

e. $t^2 \bullet (t^7)^{-2}$

$t^2 \cdot t^{-14}$

t^{-12}

$\frac{1}{t^{12}}$

f. $b^2(b^3)^{-2}$

$b^2 \cdot b^{-6}$

b^{-4}

$\frac{1}{b^4}$

Power of Product:

Original Problem: $(5ab^2)^3$ = original problem

Expanded Problem: = $5ab^2 \cdot 5ab^2 \cdot 5ab^2$

Reorganized: = $5 \cdot 5 \cdot 5 \cdot a \cdot a \cdot a \cdot b^2 \cdot b^2 \cdot b^2$

Simplified: = $5^3 \cdot a^3 \cdot b^6$

Question: What is the rule for simplifying products of powers?

$3 \cdot 2 = 6 \quad 2 \cdot 3 = 6$

Power of a Product: $(ax^m)^n = a^n \cdot x^{m \cdot n}$

Ex. 3: Raising a product to a power

a. $(3^2xy^{-6})^2$

$3^{2\cdot2}x^{1\cdot2}y^{-6\cdot2}$

$3^4\cdot x^2y^{-12} \rightarrow \frac{3^4\cdot x^2}{y^{12}}$

b. $(4g^5)^{-2}$

$4^{-2}g^{-10}$

$\frac{1}{4^2g^{10}}$

c. $(2x^{-3}yz^5)^{-4}$

$2^{-4}x^{12}y^{-4}z^{-20}$

$\frac{1\cdot x^{12}}{2^4y^4z^{20}}$

d. $(2mn)^3(3m^4)^2$

$2^3m^3n^3\cdot3^2m^8$

$2^3\cdot3^2m^{11}\cdot n^3$

$72\cdot m^{11}\cdot n^3$

e. $x^3(x^{-3}y^2z)^4$

$x^3\cdot x^{-12}\cdot y^8\cdot z^4$

$x^{-9}y^8z^4 = \frac{y^8z^4}{x^9}$

f. $(4\times10^{-3})^3$

4×10^{-9}

$\frac{1}{4\times10^9}$

The ideas behind simplifying Powers of Products also work for Powers of Quotients.

Quotient to a Power: $\left(\frac{x}{y}\right)^3 = \frac{x}{y} \cdot \frac{x}{y} \cdot \frac{x}{y} = \frac{x^3}{y^3}$

Quotient to a Power:

$$\left(\frac{x}{y}\right)^m = \frac{x^m}{y^m}$$

$$(x \cdot y^{-1})^m$$

$$x^m \cdot y^{-m}$$

$$\frac{x^m}{y^m}$$

Ex. 4: Raising a Quotient to a Power

a. $\left(\frac{m}{n}\right)^5$ **b.** $\left(\frac{3}{x^2}\right)^4$ **c.** $\left(\frac{1}{2}\right)^{-3}$

d. $\left(\frac{-2a}{b}\right)^{-2}$ **e.** $\left(\frac{8p^6q^2}{12pq^{-2}}\right)^4$