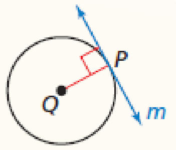
Theorems

Theorem 10.1 Tangent Line to Circle Theorem

In a plane, a line is tangent to a circle if and only if the line is perpendicular to a radius of the circle at its endpoint on the circle.

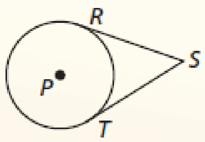


Line m is tangent to $\odot Q$ if and only if $m \perp \overline{QP}$.

Proof Ex. 47, p. 536

Theorem 10.2 External Tangent Congruence Theorem

Tangent segments from a common external point are congruent.



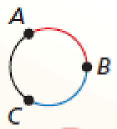
If $\overline{SR}$ and $\overline{ST}$ are tangent segments, then $\overline{SR} \cong \overline{ST}$.

Proof Ex. 46, p. 536

Postulate

Postulate 10.1 Arc Addition Postulate

The measure of an arc formed by two adjacent arcs is the sum of the measures of the two arcs.

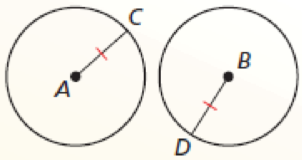


$m\overset{\frown}{ABC} = m\overset{\frown}{AB} + m\overset{\frown}{BC}$

Theorem

Theorem 10.3 Congruent Circles Theorem

Two circles are congruent circles if and only if they have the same radius.



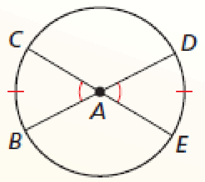
$\odot A \cong \odot B$ if and only if $\overline{AC} \cong \overline{BD}$.

Proof Ex. 35, p. 544

Theorem

Theorem 10.4 Congruent Central Angles Theorem

In the same circle, or in congruent circles, two minor arcs are congruent if and only if their corresponding central angles are congruent.



$\overset{\frown}{BC} \cong \overset{\frown}{DE}$ if and only if $\angle BAC \cong \angle DAE$.

Proof Ex. 37, p. 544

Theorem

Theorem 10.5 Similar Circles Theorem

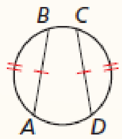
All circles are similar.

Proof p. 541; Ex. 33, p. 544

Theorems

Theorem 10.6 Congruent Corresponding Chords Theorem

In the same circle, or in congruent circles, two minor arcs are congruent if and only if their corresponding chords are congruent.

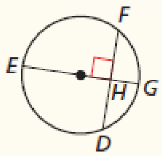


Proof Ex. 19, p. 550

$\overset{\frown}{AB} \cong \overset{\frown}{CD}$ if and only if $\overline{AB} \cong \overline{CD}$.

Theorem 10.7 Perpendicular Chord Bisector Theorem

If a diameter of a circle is perpendicular to a chord, then the diameter bisects the chord and its arc.

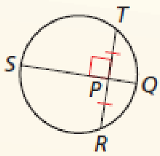


Proof Ex. 22, p. 550

If $\overline{EG}$ is a diameter and $\overline{EG} \perp \overline{DF}$, then $\overline{HD} \cong \overline{HF}$ and $\overset{\frown}{GD} \cong \overset{\frown}{GF}$.

Theorem 10.8 Perpendicular Chord Bisector Converse

If one chord of a circle is a perpendicular bisector of another chord, then the first chord is a diameter.



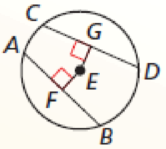
Proof Ex. 23, p. 550

If $\overline{QS}$ is a perpendicular bisector of $\overline{TR}$, then $\overline{QS}$ is a diameter of the circle.

Theorem

Theorem 10.9 Equidistant Chords Theorem

In the same circle, or in congruent circles, two chords are congruent if and only if they are equidistant from the center.



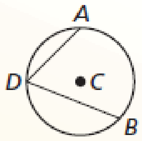
Proof Ex. 25, p. 550

$\overline{AB} \cong \overline{CD}$ if and only if $EF = EG$.

Theorem

Theorem 10.10 Measure of an Inscribed Angle Theorem

The measure of an inscribed angle is one-half the measure of its intercepted arc.



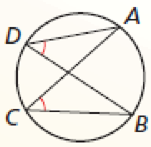
$m\angle ADB = \frac{1}{2} m\overset{\frown}{AB}$

Proof Ex. 37, p. 560

Theorem

Theorem 10.11 Inscribed Angles of a Circle Theorem

If two inscribed angles of a circle intercept the same arc, then the angles are congruent.



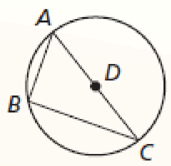
$\angle ADB \cong \angle ACB$

Proof Ex. 38, p. 560

Theorems

Theorem 10.12 Inscribed Right Triangle Theorem

If a right triangle is inscribed in a circle, then the hypotenuse is a diameter of the circle. Conversely, if one side of an inscribed triangle is a diameter of the circle, then the triangle is a right triangle and the angle opposite the diameter is the right angle.

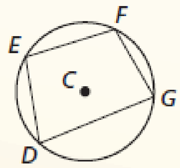


$m\angle ABC = 90°$ if and only if $\overline{AC}$ is a diameter of the circle.

Proof Ex. 39, p. 560

Theorem 10.13 Inscribed Quadrilateral Theorem

A quadrilateral can be inscribed in a circle if and only if its opposite angles are supplementary.



D, E, F, and G lie on $\odot C$ if and only if $m\angle D + m\angle F = m\angle E + m\angle G = 180°$.

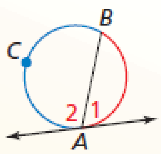
Proof Ex. 40, p. 560

Finding Angle and Arc Measures

Theorem

Theorem 10.14 Tangent and Intersected Chord Theorem

If a tangent and a chord intersect at a point on a circle, then the measure of each angle formed is one-half the measure of its intercepted arc.



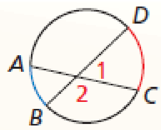
Proof Ex. 33, p. 568

$m\angle 1 = \frac{1}{2}m\overset{\frown}{AB}$ $m\angle 2 = \frac{1}{2}m\overset{\frown}{BCA}$

Theorems

Theorem 10.15 Angles Inside the Circle Theorem

If two chords intersect *inside* a circle, then the measure of each angle is one-half the *sum* of the measures of the arcs intercepted by the angle and its vertical angle.



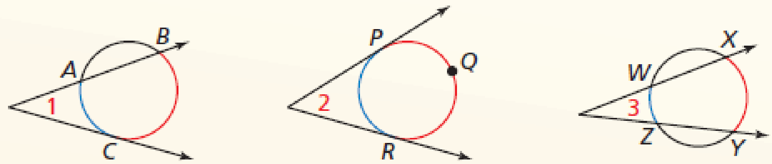
$m\angle 1 = \frac{1}{2}(m\widehat{DC} + m\widehat{AB})$,

$m\angle 2 = \frac{1}{2}(m\widehat{AD} + m\widehat{BC})$

Proof Ex. 35, p. 568

Theorem 10.16 Angles Outside the Circle Theorem

If a tangent and a secant, two tangents, or two secants intersect *outside* a circle, then the measure of the angle formed is one-half the *difference* of the measures of the intercepted arcs.



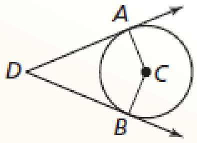
$m\angle 1 = \frac{1}{2}(m\widehat{BC} - m\widehat{AC})$ $m\angle 2 = \frac{1}{2}(m\widehat{PQR} - m\widehat{PR})$ $m\angle 3 = \frac{1}{2}(m\widehat{XY} - m\widehat{WZ})$

Proof Ex. 37, p. 568

Theorem

Theorem 10.17 Circumscribed Angle Theorem

The measure of a circumscribed angle is equal to 180° minus the measure of the central angle that intercepts the same arc.



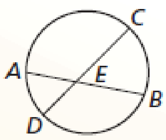
$m\angle ADB = 180° - m\angle ACB$

Proof Ex. 38, p. 568

Theorem

Theorem 10.18 Segments of Chords Theorem

If two chords intersect in the interior of a circle, then the product of the lengths of the segments of one chord is equal to the product of the lengths of the segments of the other chord.



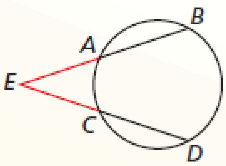
$EA \cdot EB = EC \cdot ED$

Proof Ex. 19, p. 574

Theorem

Theorem 10.19 Segments of Secants Theorem

If two secant segments share the same endpoint outside a circle, then the product of the lengths of one secant segment and its external segment equals the product of the lengths of the other secant segment and its external segment.



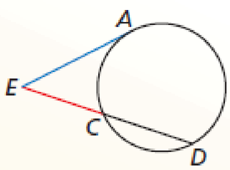
$EA \cdot EB = EC \cdot ED$

Proof Ex. 20, p. 574

Theorem

Theorem 10.20 Segments of Secants and Tangents Theorem

If a secant segment and a tangent segment share an endpoint outside a circle, then the product of the lengths of the secant segment and its external segment equals the square of the length of the tangent segment.



$EA^2 = EC \cdot ED$

Proof Exs. 21 and 22, p. 574

Core Concept

Standard Equation of a Circle

Let (x, y) represent any point on a circle with center (h, k) and radius r. By the Pythagorean Theorem (Theorem 9.1),

$$(x - h)^2 + (y - k)^2 = r^2.$$

This is the **standard equation of a circle** with center (h, k) and radius r.