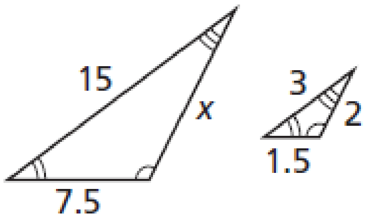
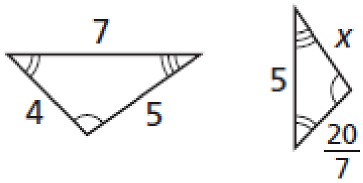
Given that the triangles are similar, find the missing side length.

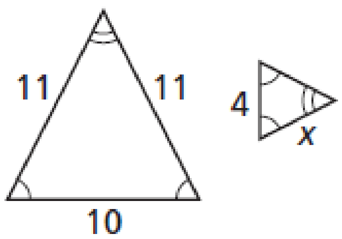
1.



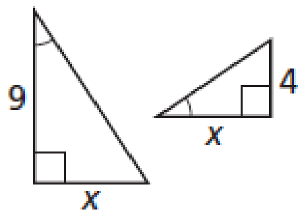
2.



3.

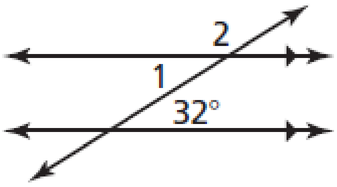


4.

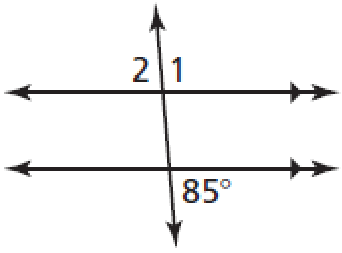


Use the diagram to find $m\angle 1$ and $m\angle 2$.

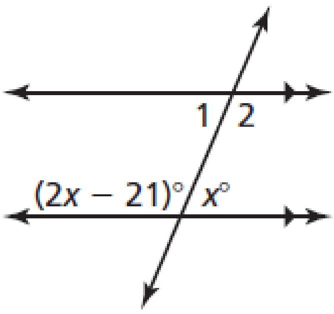
1.



2.



3.

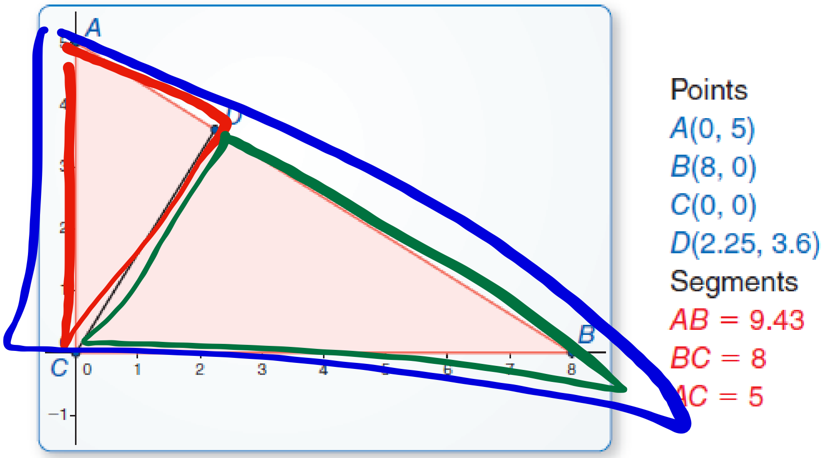


Essential Question

How are altitudes and geometric means of right triangles related?

Work with a partner.

a. Use dynamic geometry software to construct right $\triangle ABC$, as shown. Draw $\overline{CD}$ so that it is an altitude from the right angle to the hypotenuse of $\triangle ABC$.



b. The **geometric mean** of two positive numbers *a* and *b* is the positive number *x* that satisfies

$$\frac{a}{x} = \frac{x}{b}.$$

x is the geometric mean of *a* and *b*.

Write a proportion involving the side lengths of $\triangle CBD$ and $\triangle ACD$ so that *CD* is the geometric mean of two of the other side lengths. Use similar triangles to justify your steps.

c. Use the proportion you wrote in part (b) to find *CD*.

d. Generalize the proportion you wrote in part (b). Then write a conjecture about how the geometric mean is related to the altitude from the right angle to the hypotenuse of a right triangle.

$\frac{a+b}{2}$ $\sqrt{a \cdot b}$

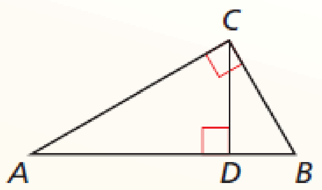
Work with a partner. Use a spreadsheet to fi nd the arithmetic mean and the geometric mean of several pairs of positive numbers. Compare the two means. What do you notice?

	A	B	C	D
1	a	b	Arithmetic Mean	Geometric Mean
2	3	4	3.5	3.464
3	4	5	4.5	4.47
4	6	7		
5	0.5	0.5		
6	0.4	0.8		
7	2	5		
8	1	4		
9	9	16		
10	10	100		
11				

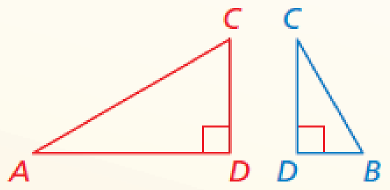
Theorem

Theorem 9.6 Right Triangle Similarity Theorem

If the altitude is drawn to the hypotenuse of a right triangle, then the two triangles formed are similar to the original triangle and to each other.



$\triangle CBD \sim \triangle ABC$, $\triangle ACD \sim \triangle ABC$, and $\triangle CBD \sim \triangle ACD$.



Proof Ex. 45, p. 484

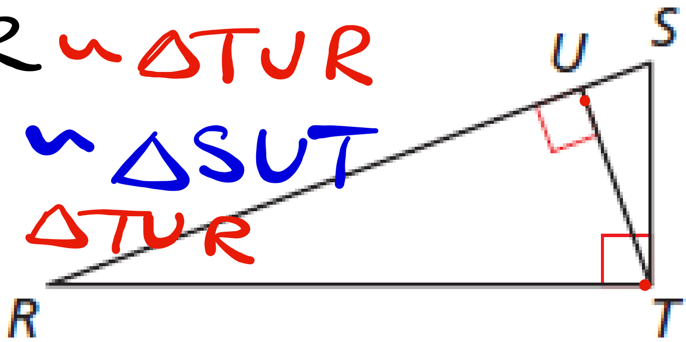
Identify the similar triangles in the diagram.

$\overline{TU}$ is the alt.

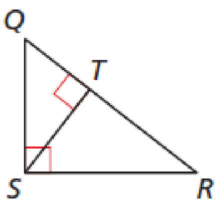
$\triangle STR \sim \triangle TUR$

$\triangle STR \sim \triangle SUT$

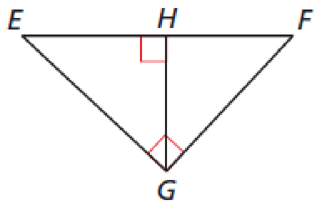
$\triangle SUT \sim \triangle TUR$



Identify the similar triangles.

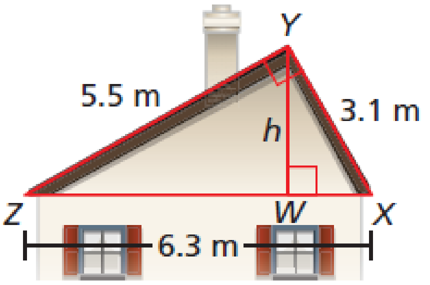
1. 

△QSR ~ △QTS ~ △STR

2. 

△EGF ~ △EHG ~ △FGH

A roof has a cross section that is a right triangle. The diagram shows the approximate dimensions of this cross section. Find the height *h* of the roof.



△ZYX ~
△ZWY ~
△YWX

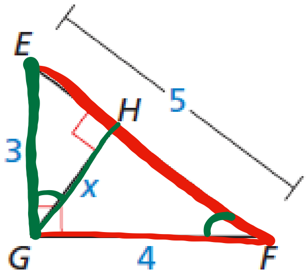
$$\frac{5.5}{6.3} = \frac{h}{3.1}$$

$$6.3h = 3.1 \cdot 5.5$$

$$h = 2.7\text{ m}$$

Find the value of *x*.

3.

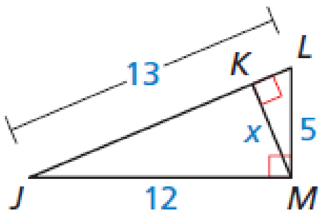


$\frac{4}{5} = \frac{x}{3}$

$12 = 5x$

$x = 2.4$

4.



$\frac{12}{13} = \frac{x}{5}$

$60 = 13x$

$x = 4.6$

Core Concept

Geometric Mean

The **geometric mean** of two positive numbers a and b is the positive number x that satisfies $\frac{a}{x} = \frac{x}{b}$. So, $x^2 = ab$ and $x = \sqrt{ab}$.

Find the geometric mean of 24 and 48.

$a = 24 \quad b = 48$

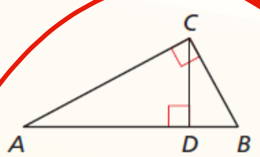
$x = \sqrt{24 \cdot 48} = \sqrt{1152} = \boxed{33.94}$

Theorems

Theorem 9.7 Geometric Mean (Altitude) Theorem

In a right triangle, the altitude from the right angle to the hypotenuse divides the hypotenuse into two segments.

The length of the altitude is the geometric mean of the lengths of the two segments of the hypotenuse.



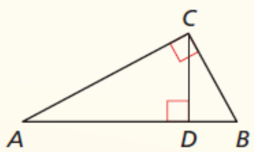
$CD^2 = AD \cdot BD$

Proof Ex. 41, p. 484

Theorem 9.8 Geometric Mean (Leg) Theorem

In a right triangle, the altitude from the right angle to the hypotenuse divides the hypotenuse into two segments.

The length of each leg of the right triangle is the geometric mean of the lengths of the hypotenuse and the segment of the hypotenuse that is adjacent to the leg.



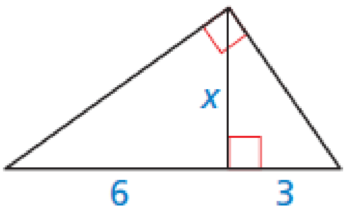
$CB^2 = DB \cdot AB$

$AC^2 = AD \cdot AB$

Proof Ex. 42, p. 484

Find the value of each variable.

a.

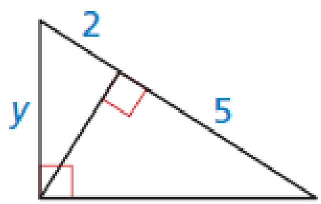


$x = \sqrt{6 \cdot 3}$

$x = \sqrt{18} = 3\sqrt{2}$

$x = 4.2$

b.

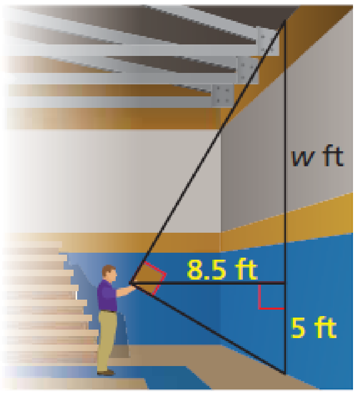


$y = \sqrt{2 \cdot 7}$

$y = \sqrt{14}$

$y = 3.74$

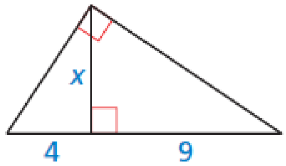
To find the cost of installing a rock wall in your school gymnasium, you need to find the height of the gym wall. You use a cardboard square to line up the top and bottom of the gym wall. Your friend measures the vertical distance from the ground to your eye and the horizontal distance from you to the gym wall. Approximate the height of the gym wall.



Find the geometric mean of the two numbers.

5. 12 and 27 **6.** 18 and 54 **7.** 16 and 18

8. Find the value of *x* in the triangle at the left.



9. WHAT IF? In Example 5, the vertical distance from the ground to your eye is 5.5 feet and the distance from you to the gym wall is 9 feet. Approximate the height of the gym wall.

Point of Most Significance: Ask students to identify, aloud or on a paper, the most significant point (or part) in the lesson that aided their learning.