Solve the equation.

1. $4^2 + 3^2 = x^2$

2. $13^2 + x^2 = 25^2$

3. $\left(\frac{5}{2}\right)^2 + x^2 = \left(\frac{1}{3}\right)^2$

4. $\left(9\sqrt{3}\right)^2 - x^2 = 2^2$

5. $\left(\sqrt{5}\right)^2 + x^2 = 12^2$

6. $\left(5\sqrt{10}\right)^2 - \left(\sqrt{2}\right)^2 = x^2$

Essential Question

How can you prove the Pythagorean Theorem?

Theorem

Theorem 9.1 Pythagorean Theorem

In a right triangle, the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the legs.

Proof Explorations 1 and 2, p. 463; Ex. 39, p. 484

$c^2 = a^2 + b^2$

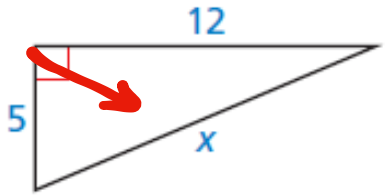
Core Concept

Common Pythagorean Triples and Some of Their Multiples

3, 4, 5	**5, 12, 13**	**8, 15, 17**	**7, 24, 25**
6, 8, 10	10, 24, 26	16, 30, 34	14, 48, 50
9, 12, 15	15, 36, 39	24, 45, 51	21, 72, 75
$3x, 4x, 5x$	$5x, 12x, 13x$	$8x, 15x, 17x$	$7x, 24x, 25x$

The most common Pythagorean triples are in bold. The other triples are the result of multiplying each integer in a bold-faced triple by the same factor.

Find the value of *x*. Then tell whether the side lengths form a Pythagorean triple.



$5^2 + 12^2 = x^2$

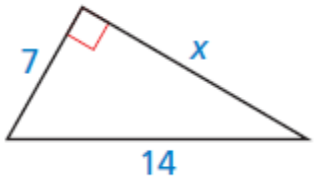
$\sqrt{x^2} = \sqrt{25 + 144}$

$x = \sqrt{25+144} = \sqrt{169}$

$x = 13$

5, 12, 13

Find the value of *x*. Then tell whether the side lengths form a Pythagorean triple.



$7^2 + x^2 = 14^2$

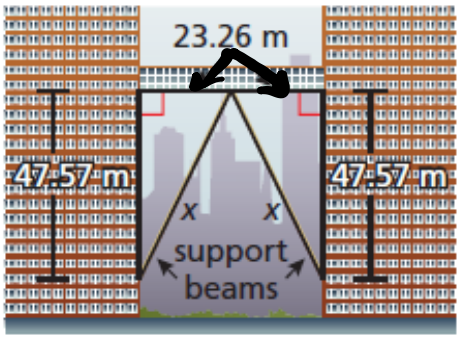
$49 + x^2 = 196$

$-49 \quad -49$

$\sqrt{x^2} = \sqrt{147}$

$x = 12.12$

The skyscrapers shown are connected by a skywalk with support beams. Use the Pythagorean Theorem to approximate the length of each support beam.



23.26m

47.57m

$23.26^2 + 47.57^2 = x^2$

$541.+2262.9 = x^2$

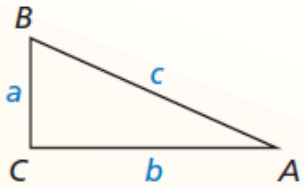
$x^2 = \sqrt{2803.9}$

$x = 52.95$ m

Theorem

Theorem 9.2 Converse of the Pythagorean Theorem

If the square of the length of the longest side of a triangle is equal to the sum of the squares of the lengths of the other two sides, then the triangle is a right triangle.

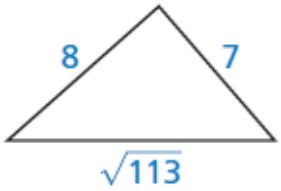


If $c^2 = a^2 + b^2$, then $\triangle ABC$ is a right triangle.

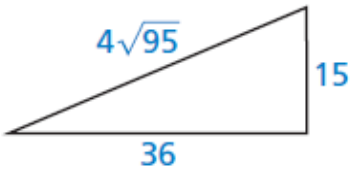
Proof Ex. 39, p. 470

Tell whether each triangle is a right triangle.

a.



b.

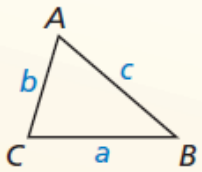


Theorem

Theorem 9.3 Pythagorean Inequalities Theorem

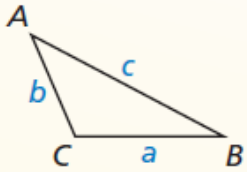
For any $\triangle ABC$, where c is the length of the longest side, the following statements are true.

If $c^2 < a^2 + b^2$, then $\triangle ABC$ is acute.



$c^2 < a^2 + b^2$

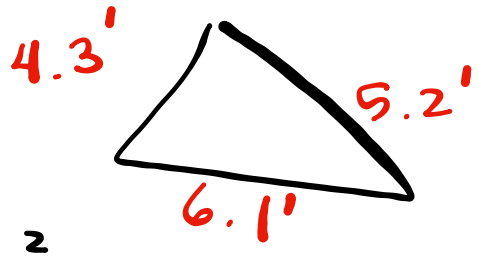
If $c^2 > a^2 + b^2$, then $\triangle ABC$ is obtuse.



$c^2 > a^2 + b^2$

Proof Exs. 42 and 43, p. 470

Verify that segments with lengths of 4.3 feet, 5.2 feet, and 6.1 feet form a triangle. Is the triangle *acute*, *right*, or *obtuse*?



$6.1^2 \_\_\_ 4.3^2 + 5.2^2$

$37.21 < 45.53$