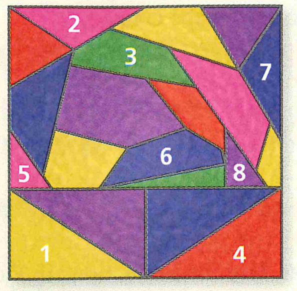
30. **HOW DO YOU SEE IT?** What type of congruence transformation can be used to verify each statement about the stained glass window?



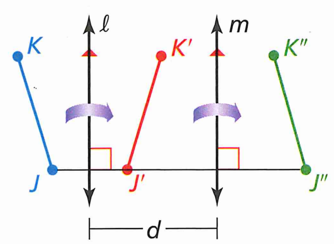
a. Triangle 5 is congruent to Triangle 8.

b. Triangle 1 is congruent to Triangle 4.

c. Triangle 2 is congruent to Triangle 7.

d. Pentagon 3 is congruent to Pentagon 6.

31. **PROVING A THEOREM** Prove the Reflections in Parallel Lines Theorem (Theorem 4.2).



Given A reflection in line ℓ maps $\overline{JK}$ to $\overline{J'K'}$, a reflection in line m maps $\overline{J'K'}$ to $\overline{J''K''}$, and $\ell \parallel m$.

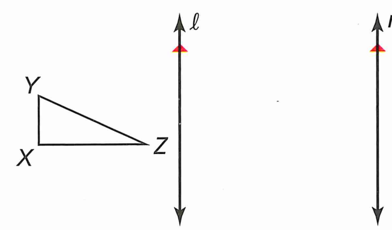
Prove a. $\overline{KK''}$ is perpendicular to ℓ and m.

b. $KK'' = 2d$, where d is the distance between ℓ and m.

32. **THOUGHT PROVOKING** A *tessellation* is the covering of a plane with congruent figures so that there are no gaps or overlaps (see Exercise 24). Draw a tessellation that involves two or more types of transformations. Describe the transformations that are used to create the tessellation.

33. **MAKING AN ARGUMENT** $\overline{PQ}$, with endpoints $P(1, 3)$ and $Q(3, 2)$, is reflected in the y-axis. The image $\overline{P'Q'}$ is then reflected in the x-axis to produce the image $\overline{P''Q''}$. One classmate says that $\overline{PQ}$ is mapped to $\overline{P''Q''}$ by the translation $(x, y) \rightarrow (x - 4, y - 5)$. Another classmate says that $\overline{PQ}$ is mapped to $\overline{P''Q''}$ by a $(2 \cdot 90)°$, or 180°, rotation about the origin. Which classmate is correct? Explain your reasoning.

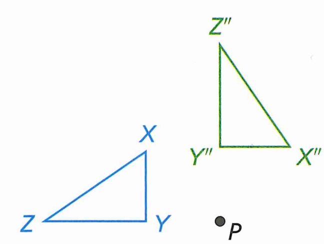
34. **CRITICAL THINKING** Does the order of reflections for a composition of two reflections in parallel lines matter? For example, is reflecting $\triangle XYZ$ in line ℓ and then its image in line m the same as reflecting $\triangle XYZ$ in line m and then its image in line ℓ?



CONSTRUCTION In Exercises 35 and 36, copy the figure. Then use a compass and straightedge to construct two lines of reflection that produce a composition of reflections resulting in the same image as the given transformation.

35. **Translation:** $\triangle ABC \rightarrow \triangle A''B''C''$

36. **Rotation about** P**:** $\triangle XYZ \rightarrow \triangle X''Y''Z''$



Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Solve the equation. Check your solution. *(Skills Review Handbook)*

37. $5x + 16 = -3x$

38. $12 + 6m = 2m$

39. $4b + 8 = 6b - 4$

40. $7w - 9 = 13 - 4w$

41. $7(2n + 11) = 4n$

42. $-2(8 - y) = -6y$

43. Last year, the track team's yard sale earned \$500. This year, the yard sale earned \$625. What is the percent of increase? *(Skills Review Handbook)*

4.5 Dilations

COMMON CORE

Learning Standards
HSG-CO.A.2
HSG-SRT.A.1a
HSG-SRT.A.1b

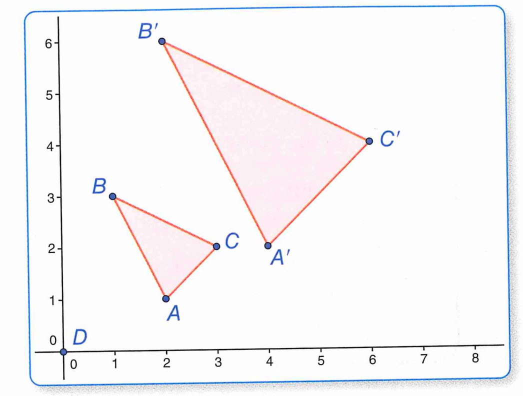
Essential Question

What does it mean to dilate a figure?

EXPLORATION 1 Dilating a Triangle in a Coordinate Plane

Work with a partner. Use dynamic geometry software to draw any triangle and label it $\triangle ABC$.

a. *Dilate* $\triangle ABC$ using a *scale factor* of 2 and a *center of dilation* at the origin to form $\triangle A'B'C'$. Compare the coordinates, side lengths, and angle measures of $\triangle ABC$ and $\triangle A'B'C'$.



Sample

Points
$A(2, 1)$
$B(1, 3)$
$C(3, 2)$

Segments
$AB = 2.24$
$BC = 2.24$
$AC = 1.41$

Angles
$m\angle A = 71.57°$
$m\angle B = 36.87°$
$m\angle C = 71.57°$

b. Repeat part (a) using a *scale factor* of $\frac{1}{2}$.

c. What do the results of parts (a) and (b) suggest about the coordinates, side lengths, and angle measures of the image of $\triangle ABC$ after a dilation with a scale factor of k?

LOOKING FOR STRUCTURE

To be proficient in math, you need to look closely to discern a pattern or structure.

EXPLORATION 2 Dilating Lines in a Coordinate Plane

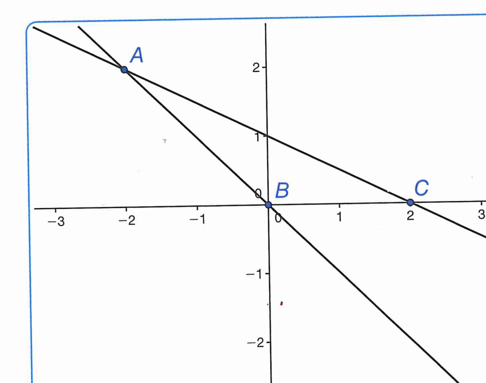
Work with a partner. Use dynamic geometry software to draw $\overleftrightarrow{AB}$ that passes through the origin and $\overleftrightarrow{AC}$ that does not pass through the origin.

a. *Dilate* $\overleftrightarrow{AB}$ using a *scale factor* of 3 and a *center of dilation* at the origin. Describe the image.

b. *Dilate* $\overleftrightarrow{AC}$ using a *scale factor* of 3 and a *center of dilation* at the origin. Describe the image.

c. Repeat parts (a) and (b) using a scale factor of $\frac{1}{4}$.

d. What do you notice about dilations of lines passing through the center of dilation and dilations of lines not passing through the center of dilation?



Sample	Points	Lines
	$A(-2, 2)$	$x + y = 0$
	$B(0, 0)$	$x + 2y = 2$
	$C(2, 0)$	

Communicate Your Answer

3. What does it mean to dilate a figure?

4. Repeat Exploration 1 using a center of dilation at a point other than the origin.

4.5 Lesson

Core Vocabulary

dilation, *p. 208*
center of dilation, *p. 208*
scale factor, *p. 208*
enlargement, *p. 208*
reduction, *p. 208*

What You Will Learn

- Identify and perform dilations.
- Solve real-life problems involving scale factors and dilations.

Identifying and Performing Dilations

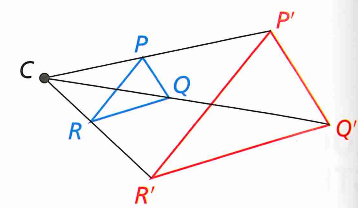
Core Concept

Dilations

A **dilation** is a transformation in which a figure is enlarged or reduced with respect to a fixed point C called the **center of dilation** and a **scale factor** k, which is the ratio of the lengths of the corresponding sides of the image and the preimage.

A dilation with center of dilation C and scale factor k maps every point P in a figure to a point P' so that the following are true.

- If P is the center point C, then $P = P'$.
- If P is not the center point C, then the image point P' lies on $\overrightarrow{CP}$. The scale factor k is a positive number such that $k = \frac{CP'}{CP}$.
- Angle measures are preserved.



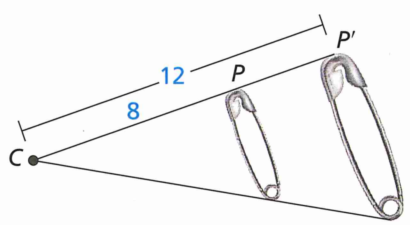
A dilation does not change any line that passes through the center of dilation. A dilation maps a line that does not pass through the center of dilation to a parallel line. In the figure above, $\overleftrightarrow{PR} \parallel \overleftrightarrow{P'R'}$, $\overleftrightarrow{PQ} \parallel \overleftrightarrow{P'Q'}$, and $\overleftrightarrow{QR} \parallel \overleftrightarrow{Q'R'}$.

When the scale factor $k > 1$, a dilation is an **enlargement**. When $0 < k < 1$, a dilation is a **reduction**.

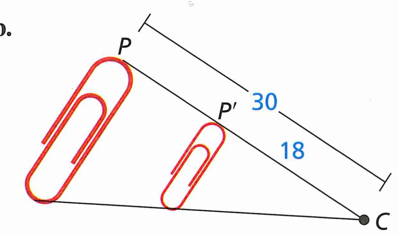
EXAMPLE 1 Identifying Dilations

Find the scale factor of the dilation. Then tell whether the dilation is a *reduction* or an *enlargement*.

a.



b.



READING

The scale factor of a dilation can be written as a fraction, decimal, or percent.

SOLUTION

a. Because $\frac{CP'}{CP} = \frac{12}{8}$, the scale factor is $k = \frac{3}{2}$. So, the dilation is an enlargement.

b. Because $\frac{CP'}{CP} = \frac{18}{30}$, the scale factor is $k = \frac{3}{5}$. So, the dilation is a reduction.

Monitoring Progress

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1. In a dilation, $CP' = 3$ and $CP = 12$. Find the scale factor. Then tell whether the dilation is a *reduction* or an *enlargement*.

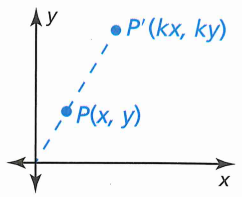
READING DIAGRAMS

In this chapter, for all of the dilations in the coordinate plane, the center of dilation is the origin unless otherwise noted.

Core Concept

Coordinate Rule for Dilations

If $P(x, y)$ is the preimage of a point, then its image after a dilation centered at the origin $(0, 0)$ with scale factor k is the point $P'(kx, ky)$.



$(x, y) \rightarrow (kx, ky)$

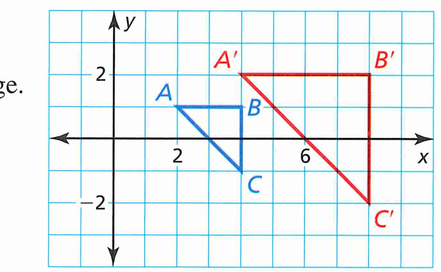
EXAMPLE 2 Dilating a Figure in the Coordinate Plane

Graph $\triangle ABC$ with vertices $A(2, 1)$, $B(4, 1)$, and $C(4, -1)$ and its image after a dilation with a scale factor of 2.

SOLUTION

Use the coordinate rule for a dilation with $k = 2$ to find the coordinates of the vertices of the image. Then graph $\triangle ABC$ and its image.

$$(x, y) \rightarrow (2x, 2y)$$
$$A(2, 1) \rightarrow A'(4, 2)$$
$$B(4, 1) \rightarrow B'(8, 2)$$
$$C(4, -1) \rightarrow C'(8, -2)$$



Notice the relationships between the lengths and slopes of the sides of the triangles in Example 2. Each side length of $\triangle A'B'C'$ is longer than its corresponding side by the scale factor. The corresponding sides are parallel because their slopes are the same.

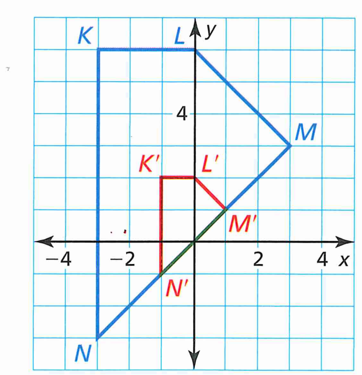
EXAMPLE 3 Dilating a Figure in the Coordinate Plane

Graph quadrilateral $KLMN$ with vertices $K(-3, 6)$, $L(0, 6)$, $M(3, 3)$, and $N(-3, -3)$ and its image after a dilation with a scale factor of $\frac{1}{3}$.

SOLUTION

Use the coordinate rule for a dilation with $k = \frac{1}{3}$ to find the coordinates of the vertices of the image. Then graph quadrilateral $KLMN$ and its image.

$$(x, y) \rightarrow \left(\tfrac{1}{3}x, \tfrac{1}{3}y\right)$$
$$K(-3, 6) \rightarrow K'(-1, 2)$$
$$L(0, 6) \rightarrow L'(0, 2)$$
$$M(3, 3) \rightarrow M'(1, 1)$$
$$N(-3, -3) \rightarrow N'(-1, -1)$$



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Graph $\triangle PQR$ and its image after a dilation with scale factor k.

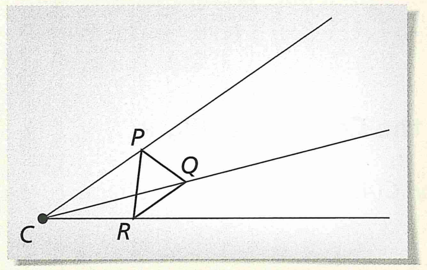
2. $P(-2, -1)$, $Q(-1, 0)$, $R(0, -1)$; $k = 4$
3. $P(5, -5)$, $Q(10, -5)$, $R(10, 5)$; $k = 0.4$

CONSTRUCTION Constructing a Dilation

Use a compass and straightedge to construct a dilation of $\triangle PQR$ with a scale factor of 2. Use a point C outside the triangle as the center of dilation.

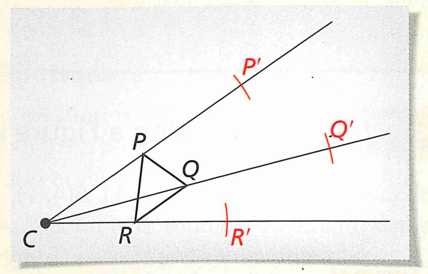
SOLUTION

Step 1



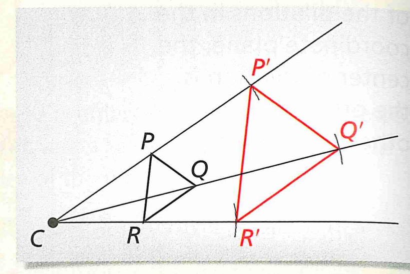
Draw a triangle Draw $\triangle PQR$ and choose the center of the dilation C outside the triangle. Draw rays from C through the vertices of the triangle.

Step 2

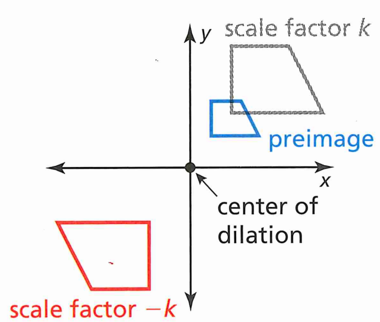


Use a compass Use a compass to locate P' on $\overrightarrow{CP}$ so that $CP' = 2(CP)$. Locate Q' and R' using the same method.

Step 3



Connect points Connect points P', Q', and R' to form $\triangle P'Q'R'$.



In the coordinate plane, you can have scale factors that are negative numbers. When this occurs, the figure rotates 180°. So, when $k > 0$, a dilation with a scale factor of $-k$ is the same as the composition of a dilation with a scale factor of k followed by a rotation of 180° about the center of dilation. Using the coordinate rules for a dilation and a rotation of 180°, you can think of the notation as

$$(x, y) \rightarrow (kx, ky) \rightarrow (-kx, -ky).$$

EXAMPLE 4 Using a Negative Scale Factor

Graph $\triangle FGH$ with vertices $F(-4, -2)$, $G(-2, 4)$, and $H(-2, -2)$ and its image after a dilation with a scale factor of $-\frac{1}{2}$.

SOLUTION

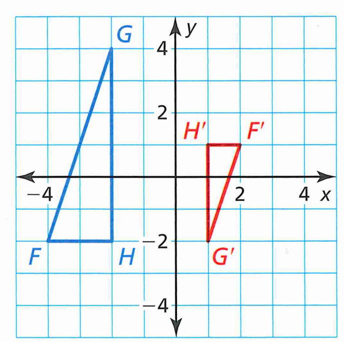
Use the coordinate rule for a dilation with $k = -\frac{1}{2}$ to find the coordinates of the vertices of the image. Then graph $\triangle FGH$ and its image.

$$(x, y) \rightarrow \left(-\tfrac{1}{2}x, -\tfrac{1}{2}y\right)$$

$F(-4, -2) \rightarrow F'(2, 1)$

$G(-2, 4) \rightarrow G'(1, -2)$

$H(-2, -2) \rightarrow H'(1, 1)$



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4. Graph $\triangle PQR$ with vertices $P(1, 2)$, $Q(3, 1)$, and $R(1, -3)$ and its image after a dilation with a scale factor of -2.
5. Suppose a figure containing the origin is dilated. Explain why the corresponding point in the image of the figure is also the origin.

Solving Real-Life Problems

READING

Scale factors are written so that the units in the numerator and denominator divide out.

EXAMPLE 5 Finding a Scale Factor

You are making your own photo stickers. Your photo is 4 inches by 4 inches. The image on the stickers is 1.1 inches by 1.1 inches. What is the scale factor of this dilation?

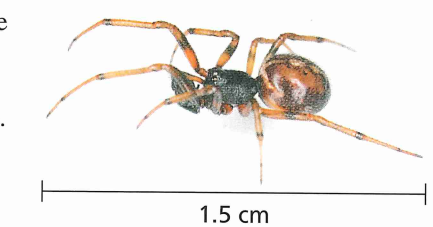
SOLUTION

The scale factor is the ratio of a side length of the sticker image to a side length of the original photo, or $\frac{1.1 \text{ in.}}{4 \text{ in.}}$.

▶ So, in simplest form, the scale factor is $\frac{11}{40}$.

EXAMPLE 6 Finding the Length of an Image

You are using a magnifying glass that shows the image of an object that is six times the object's actual size. Determine the length of the image of the spider seen through the magnifying glass.



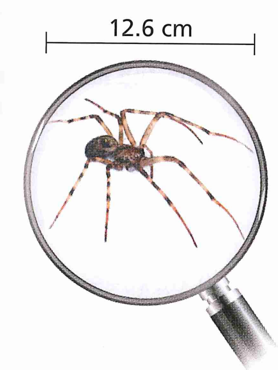
SOLUTION

$$\frac{\text{image length}}{\text{actual length}} = k$$

$$\frac{x}{1.5} = 6$$

$$x = 9$$

▶ So, the image length through the magnifying glass is 9 centimeters.



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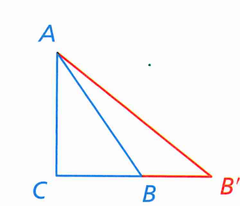
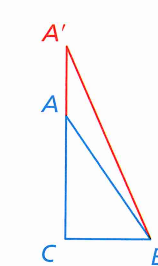
6. An optometrist dilates the pupils of a patient's eyes to get a better look at the back of the eyes. A pupil dilates from 4.5 millimeters to 8 millimeters. What is the scale factor of this dilation?
7. The image of a spider seen through the magnifying glass in Example 6 is shown at the left. Find the actual length of the spider.

When a transformation, such as a dilation, changes the shape or size of a figure, the transformation is *nonrigid*. In addition to dilations, there are many possible nonrigid transformations. Two examples are shown below. It is important to pay close attention to whether a nonrigid transformation preserves lengths and angle measures.

Horizontal Stretch



Vertical Stretch



4.5 Exercises

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Vocabulary and Core Concept Check

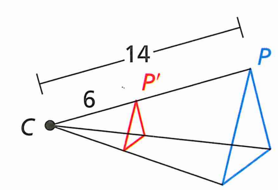
1. **COMPLETE THE SENTENCE** If $P(x, y)$ is the preimage of a point, then its image after a dilation centered at the origin $(0, 0)$ with scale factor k is the point __________.

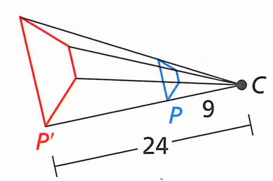
2. **WHICH ONE DOESN'T BELONG?** Which scale factor does *not* belong with the other three? Explain your reasoning.

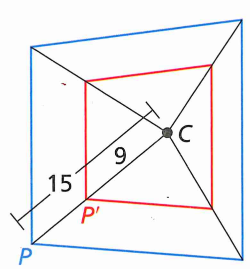
$\frac{5}{4}$	60%	115%	2

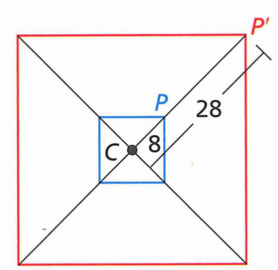
Monitoring Progress and Modeling with Mathematics

In Exercises 3–6, find the scale factor of the dilation. Then tell whether the dilation is a *reduction* or an *enlargement*. *(See Example 1.)*

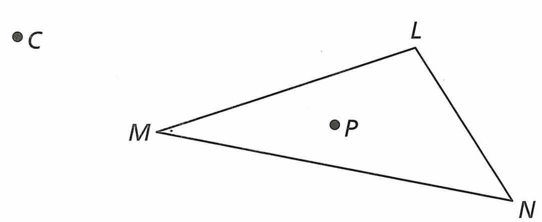
3. 

4. 

5. 

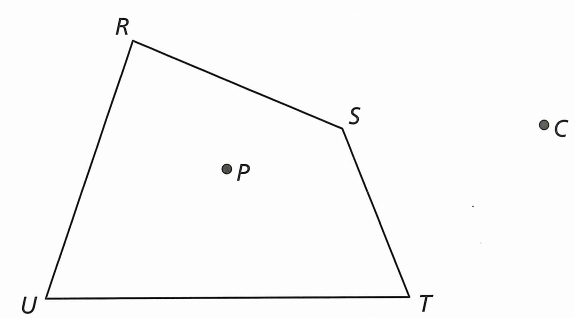
6. 

CONSTRUCTION **In Exercises 7–10, copy the diagram. Then use a compass and straightedge to construct a dilation of $\triangle LMN$ with the given center and scale factor k.**



7. Center C, $k = 2$
8. Center P, $k = 3$
9. Center M, $k = \frac{1}{2}$
10. Center C, $k = 25\%$

CONSTRUCTION **In Exercises 11–14, copy the diagram. Then use a compass and straightedge to construct a dilation of quadrilateral $RSTU$ with the given center and scale factor k.**



11. Center C, $k = 3$
12. Center P, $k = \frac{1}{3}$
13. Center R, $k = 0.25$
14. Center C, $k = 75\%$

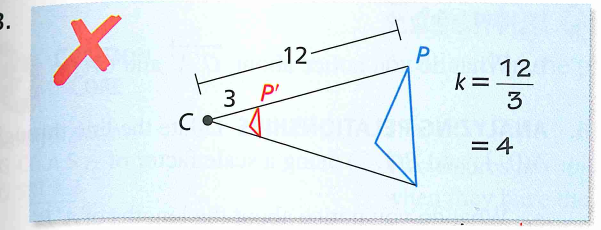
In Exercises 15–18, graph the polygon and its image after a dilation with scale factor k. *(See Examples 2 and 3.)*

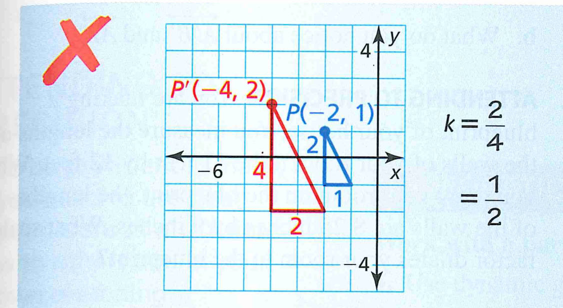
15. $X(6, -1)$, $Y(-2, -4)$, $Z(1, 2)$; $k = 3$
16. $A(0, 5)$, $B(-10, -5)$, $C(5, -5)$; $k = 120\%$
17. $T(9, -3)$, $U(6, 0)$, $V(3, 9)$, $W(0, 0)$; $k = \frac{2}{3}$
18. $J(4, 0)$, $K(-8, 4)$, $L(0, -4)$, $M(12, -8)$; $k = 0.25$

In Exercises 19–22, graph the polygon and its image after a dilation with scale factor k. *(See Example 4.)*

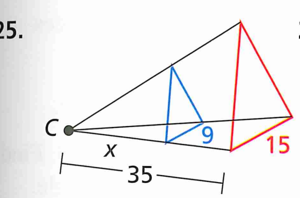
19. $B(-5, -10)$, $C(-10, 15)$, $D(0, 5)$; $k = -\frac{1}{5}$
20. $L(0, 0)$, $M(-4, 1)$, $N(-3, -6)$; $k = -3$
21. $R(-7, -1)$, $S(2, 5)$, $T(-2, -3)$, $U(-3, -3)$; $k = -4$
22. $W(8, -2)$, $X(6, 0)$, $Y(-6, 4)$, $Z(-2, 2)$; $k = -0.5$

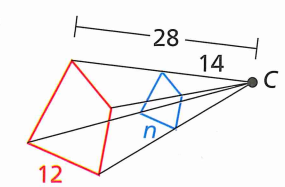
ERROR ANALYSIS **In Exercises 23 and 24, describe and correct the error in finding the scale factor of the dilation.**

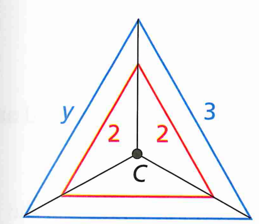
23. 

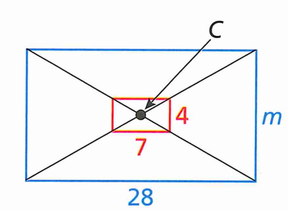
24. 

In Exercises 25–28, the red figure is the image of the blue figure after a dilation with center C. Find the scale factor of the dilation. Then find the value of the variable.

25. 

26. 

27. 

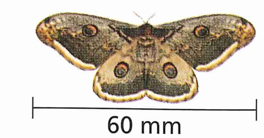
28. 

29. **FINDING A SCALE FACTOR** You receive wallet-sized photos of your school picture. The photo is 2.5 inches by 3.5 inches. You decide to dilate the photo to 5 inches by 7 inches at the store. What is the scale factor of this dilation? *(See Example 5.)*

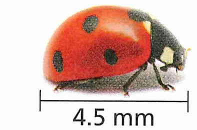
30. **FINDING A SCALE FACTOR** Your visually impaired friend asked you to enlarge your notes from class so he can study. You took notes on 8.5-inch by 11-inch paper. The enlarged copy has a smaller side with a length of 10 inches. What is the scale factor of this dilation? *(See Example 5.)*

In Exercises 31–34, you are using a magnifying glass. Use the length of the insect and the magnification level to determine the length of the image seen through the magnifying glass. *(See Example 6.)*

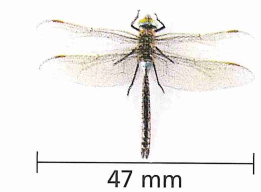
31. emperor moth
Magnification: 5×



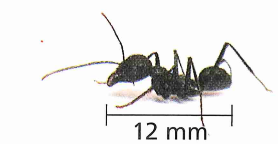
32. ladybug
Magnification: 10×



33. dragonfly
Magnification: 20×

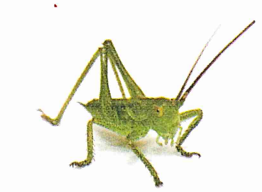


34. carpenter ant
Magnification: 15×



35. **ANALYZING RELATIONSHIPS** Use the given actual and magnified lengths to determine which of the following insects were looked at using the same magnifying glass. Explain your reasoning.

grasshopper
Actual: 2 in.
Magnified: 15 in.



black beetle
Actual: 0.6 in.
Magnified: 4.2 in.

honeybee
Actual: $\frac{5}{8}$ in.
Magnified: $\frac{75}{16}$ in.

monarch butterfly
Actual: 3.9 in.
Magnified: 29.25 in.

36. **THOUGHT PROVOKING** Draw $\triangle ABC$ and $\triangle A'B'C'$ so that $\triangle A'B'C'$ is a dilation of $\triangle ABC$. Find the center of dilation and explain how you found it.

37. **REASONING** Your friend prints a 4-inch by 6-inch photo for you from the school dance. All you have is an 8-inch by 10-inch frame. Can you dilate the photo to fit the frame? Explain your reasoning.